

# *On Some Nonlinear Boundary Problems for the Equation of Minimal Surfaces\**

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The purpose of this paper is to introduce certain nonlinear boundary value problems for the Euler minimal surfaces equation [3]; and, using only integral identity methods, to prove for each a uniqueness theorem. In Section III, a non-existence result is obtained for certain of these problems. Finally, for those problems which are solvable, the solutions are compared (Section IV) with harmonic functions solving analogous problems.

**I. Introduction.** The equation

$$(1) \quad (1 + u_y^2)u_{xx} - 2u_x u_y u_{xy} + (1 + u_x^2)u_{yy} = 0$$

can be thought of as the equation satisfied by the  $z$ -coördinate of a minimal surface  $z = u(x, y)$ , or as the equation satisfied by the potential in the "flow" of a Chaplygin gas. Although the latter correspondence is technically a fiction, since it presupposes an unreal density-speed relation  $\rho = (1 + q^2)^{-1/2}$ , it is in agreement up to terms of order  $q^4$  with the precise relation in any subsonic region [1], where the character of the equation is very similar to that of the compressible flow equation. Hence it is logical to suspect that the results established herein carry over, with suitable speed restrictions, to compressible flows; and the realization of this will be, in fact, the substance of a sequel to this paper. Preliminary results may be found in [4].

We shall consider a domain  $D$  having a piecewise-smooth boundary  $\Gamma$ . We assume that  $u_x$  and  $u_y$  are  $C'(D)$  and  $C^0(D + \Gamma)$ ; and are the single-valued partial derivatives of a function  $u(x, y)$  which, in general, is not itself required to be single-valued [1]. However, when we compare two functions  $u_1$  and  $u_2$ , it is understood that their difference is single-valued. In the case of potential flows, this amounts to fixing the circulation [4]. The outward unit normal to  $D$  will be denoted by " $n$ ," the outward normal derivative of  $u$  by " $u_n$ ," and arc length along  $\Gamma$  by " $s$ ."

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