

Infinite Particle Systems and Multi-Dimensional Renewal Theory

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Communicated by WILLIAM FELLER

1. Introduction and statement of results. The obvious extension of the main theorem of one-dimensional renewal theory to the multi-dimensional case is degenerate in that all limits are zero or infinity. In this paper we will consider multi-dimensional renewal theory in the context of an infinite particle system. By this device we will be able to obtain a non-degenerate theory which closely parallels one-dimensional renewal theory. The methods used below depend on a closely related work of the author [13] and on the main theorem of one-dimensional renewal theory, proven in its final form by Feller and Orey [6].

Let φ denote a probability measure on d -dimensional space R^d , Let $\varphi^{(n)}$ denote the n -fold convolution of φ with itself and set

$$\mu = \sum_{n=0}^{\infty} \varphi^{(n)}.$$

Then μ is the *renewal measure*.

The behavior of μ depends on the smallest closed subgroup X of R^d containing the support of φ . Without loss of generality we can assume that

$$X = \{x = (x^1, \dots, x^d) \mid x^{d_1+1}, \dots, x^d \text{ are integers}\}.$$

If $d_1 = d$, then $X = R^d$ and if $d_1 = 0$, then $X = Z^d$, where Z^d denotes the set of points in R^d all of whose coordinates are integers.

Let $| \cdot |$ denote Haar measure on X defined as the product of Lebesgue measure on the first d_1 coordinates of X and counting measure on the last $d - d_1$ coordinates. Let \mathcal{B} denote the collection of relatively compact Borel subsets of X . Let \mathcal{A} denote the subcollection of $A \in \mathcal{B}$ such that $|\partial A| = 0$.

For some results it will be necessary to assume that φ satisfies

Condition 1. *Some $\varphi^{(n)}$ is non-singular with respect to Haar measure on X .*

Let $\mathcal{A}^* = \mathcal{B}$ if φ satisfies Condition 1 and let $\mathcal{A}^* = \mathcal{A}$ otherwise.

Let \cdot denote the usual dot product on R^d and let v denote a vector in R^d of unit length. Let the non-negative constant $\kappa = \kappa_v$ be defined as follows: if

* The preparation of this paper was supported in part by NSF Grant # GP 5224.