

# *An Existence Theorem for a Class of Non-linear Integral Equations with Applications to a Non-linear Elliptic Boundary Value Problem\**

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In [5], Nehari proved that if  $\Omega$  is a bounded real interval,  $K$  a symmetric positive definite kernel which is continuous on  $\Omega \times \Omega$ , and  $F$  a non-negative continuous function on  $\bar{R}_+ \times \Omega$  satisfying, for some  $\epsilon > 0$ ,

$$(1) \quad 0 < \eta_1^{-\epsilon} F(\eta_1, x) \leq \eta_2^{-\epsilon} F(\eta_2, x), \quad 0 < \eta_1 < \eta_2 < \infty,$$

for all  $x \in \Omega$ , then the integral equation

$$(2) \quad y(x) = \int_{\Omega} K(x, t) y(t) F(y^2(t), t) dt,$$

has a non-trivial solution which is continuous on  $\Omega$ . The proof uses variational methods. In this note we shall prove, using arguments similar to those of [5], that if  $\Omega$  is a bounded region in  $R^n$  ( $n$ -dimensional Euclidean space), if  $K$  is a symmetric function on  $\Omega \times \Omega$  such that for some pair of conjugate indices  $p, q$ ,  $1 < q \leq 2 \leq p < \infty$ ,

$$(3) \quad [Au](x) = \int_{\Omega} K(x, t) u(t) dt,$$

defines a completely continuous operator  $A$  from  $L^q(\Omega)$  to  $L^p(\Omega)$ , which is positive definite in the sense that

$$\int_{\Omega} \int_{\Omega} K(x, t) u(x) u(t) dx dt > 0, \quad u \in L^q(\Omega) \setminus \{0\},$$

and if in addition to (1),  $F$  satisfies the appropriate polynomial growth inequality so that the mapping

$$y(x) \rightarrow y(x) F(y^2(x), x)$$

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