

Submersions of Codimension 1

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1. Introduction. A differentiable map $f : M \rightarrow X$ is called a *submersion* provided the differential $df : M_p \rightarrow X_{f(p)}$ is surjective for all $p \in M$. This implies of course that the dimension of M is greater than or equal to the dimension of X . Setting $r = \dim M - \dim X$ (r is the *codimension* of f) we note that $f^{-1}(x)$ constitutes a regularly imbedded r -dimensional submanifold of M for all $x \in X$. These submanifolds will be called the *fibres* of f . We shall assume that M and X are paracompact Hausdorff C^∞ -manifolds, and that X is connected. When M is compact, or more generally when the fibres of f are all compact, f is known to be a fibration (cf. [2]), and for this case one has a fairly extensive theory. On the other hand, very little is known as yet regarding submersions which are not fibrations. One notable result in this direction has recently been obtained by Anthony Phillips, who has studied the spaces of all submersions $f : M \rightarrow X$ for given pairs (M, X) (cf. [4]). In contrast to these investigations of Phillips we shall be interested to study the structure of individual submersions which map a given space M onto a target X . Moreover, the present paper confines itself to the case of codimension 1.

The fibres $f^{-1}(x)$ consist now of one or more connected components (we call these the *leaves* of f) each of which is diffeomorphic to the circle or to the real line, depending on whether it is compact or noncompact. The submersion will be called *simple* if all its fibres are connected. It will be called *orientable* if its fibres can be oriented in a continuous manner. As it turns out, the homology structure of a simple orientable f can be described in terms of a generalized Thom-Gysin sequence as follows (cf. [7]). We have learned from a private communication that the sequence given by Theorem A was found independently by Glen Bredon.)

Theorem A. *Let $f : M \rightarrow X$ be a simple orientable submersion of codimension 1, and let U denote the set of all $x \in X$ such that $f^{-1}(x)$ is compact. There exists an exact sequence*

$$\cdots \rightarrow H_q(M) \xrightarrow{f^*} H_q(X) \xrightarrow{\alpha} H_{q-2}(U) \xrightarrow{\beta} H_{q-1}(M) \rightarrow \cdots$$

relating the integral homology groups.

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