

# *An Operator Valued Function Space Integral and a Related Integral Equation*

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§0. Introduction. Let us begin by considering for positive values of  $\lambda$ ,

$$(0.1) \quad (I_\lambda(F)\psi)(\xi) \equiv \int_{C_0[a,b]} F(\lambda^{-1/2}x + \xi)\psi(\lambda^{-1/2}x(b) + \xi) dx,$$

where  $C_0[a, b] = \{x(\cdot) \mid x(a) = 0, x(t) \text{ continuous on } [a, b]\}$ , and  $F$  is a real or complex valued functional defined for all continuous functions on  $[a, b]$  and  $\psi$  is a real or complex valued function defined almost everywhere on  $(-\infty, \infty)$  and  $\xi$  is a real number; and the  $\lambda, F, \psi, \xi$  are so chosen that the above Wiener integral exists. Now suppose that for a certain choice of the  $\lambda, F, \psi$  the integral (0.1) exists for all (or almost all) values of  $\xi$  in a set  $\mathcal{S}$  of real numbers. Then

$$(0.2) \quad I_\lambda(F)\psi$$

denotes the function which maps  $\xi$  into  $(I_\lambda(F)\psi)(\xi)$  for almost all values of  $\xi$  in  $\mathcal{S}$ . Again, suppose that for a certain choice of  $\lambda, F$ , the function (0.2) exists for  $\psi$  in a class of functions  $\mathfrak{D}$  and belongs to a class of functions  $\mathcal{E}$ . Then the operator

$$(0.3) \quad I_\lambda(F)$$

maps  $\mathfrak{D}$  into  $\mathcal{E}$ . In what follows,  $\mathfrak{D}$  and  $\mathcal{E}$  will usually be  $L_2(-\infty, \infty)$ , so  $I_\lambda(F)$  will usually be an operator that maps the Hilbert space  $L_2$  into  $L_2$ . It can therefore be considered to be an operator valued function space integral. We shall be particularly interested in extending the definitions of (0.1), (0.2), (0.3) to the case where  $\lambda$  is not real, and in particular, to the case where  $\lambda$  is a pure imaginary number. Before doing so, however, we mention that if

$$(0.4) \quad F(x) = \exp \left\{ \int_a^t \theta(s, x(s)) ds \right\}$$

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