

On the Range of General Wiener–Hopf Operators

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1. Introduction. What is usually called a Wiener–Hopf equation is an equation of the form

$$(1.1) \quad \int_0^{\infty} k(x-y)f(y) dy = g(x), \quad x > 0,$$

or one of several obvious modifications of this equation, all involving an integral equation with a difference kernel, the range of integration being the half-line $(0, \infty)$.

The usual method for solving such equations involves a good deal of hard analysis and, in particular, a heavy use of complex variables. (See, *e.g.*, [3, 6].) However, as has been noted several times, and as I pointed out explicitly in [5], the method can be looked on as a special case of the method of orthogonal projection, and as such possesses a far-reaching generalization.

To understand this, let H be a Hilbert space, and let A be an operator on it. Let P be any orthogonal projection on H , let $R(P)$ denote the range of P , and let $A_{R(P)}$ denote the restriction of A to $R(P)$. We shall call any equation of the form

$$(1.2) \quad PA_{R(P)}f = Pg$$

a *Wiener–Hopf equation*. The operator $PA_{R(P)}$ will be called a *Wiener–Hopf operator*.

(1.1) is a special case of (1.2). If $H = L^2(-\infty, \infty)$, if

$$(1.3) \quad (Af)(x) = \int_{-\infty}^{\infty} k(x-y)f(y) dy,$$

and if

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