

Fundamental Solutions and Properties of Solutions of the Initial Value Radial Euler-Poisson-Darboux Problem

L. R. BRAGG*

Communicated by J. B. DIAZ

1. Introduction. In this paper, we develop fundamental solutions and examine the properties of solutions of initial value problems related to the equation

$$(1.1) \quad \frac{\partial^2 u}{\partial t^2}(r, t) + \frac{a}{t} \frac{\partial u}{\partial t}(r, t) = \frac{\partial^2 u}{\partial r^2}(r, t) + \frac{\mu - 1}{r} \frac{\partial u}{\partial r}(r, t), \quad r, t \geq 0$$

for $a \geq 0$ and $\mu \geq 1$. This is just the Euler-Poisson-Darboux equation generalized to the radially symmetric case. If $a > 0$, the appropriate initial conditions for (1.1) have the form

$$(1.2) \quad u(r, 0) = \phi(r), \quad u_t(r, 0) = 0$$

while if $a = 0$, (1.1) admits the more general conditions

$$(1.3) \quad u(r, 0) = \phi(r), \quad u_t(r, 0) = \psi(r).$$

In the latter case, (1.1) is, of course, the radial wave equation.

A number of authors, including Asgierrson [1], Blum [2], Diaz and Weinberger [7], Lieberstein [8], Weinstein [11], [12], have made important contributions in connection with the standard Euler-Poisson-Darboux equation. A more complete listing of contributors may be found in the references of [12]. Many of the properties previously developed for the solutions of the standard equation are applicable to the solutions of (1.1). These are the relations among solutions that involve at dependence (such as the Weinstein relations). If we denote a solution of (1.1) by $u^{(a, \mu)}(r, t)$, then these relations are:

$$(1.4) \quad \begin{cases} \text{(a)} & \frac{1}{2} \frac{\partial}{\partial(t^2)} u^{(a, \mu)}(r, t) = u^{(a+2, \mu)}(r, t) \\ \text{(b)} & u^{(2-a, \mu)}(r, t) = t^{a-1} u^{(a, \mu)}(r, t) \end{cases}$$

These are index shifting relations on the parameter a .

* This research was supported by N. S. F. Grant GP-7382.