

# *Fuchsian Groups of Convergence Type and Poincaré Series of Dimension $-2$*

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**§1. Introduction.** Throughout  $\Gamma$  stands for a group of Möbius transformations acting discontinuously on the unit disc  $U$  of the complex plane. We assume that  $\Gamma$  acts freely on  $U$ , i.e., that  $\Gamma$  contains no elliptic transformations. The principal object of this paper is to study certain aspects of Poincaré theta series of dimension  $-2$  associated with  $\Gamma$ , namely, series of the form  $\sum_{T \in \Gamma} f(Tz) \cdot T'(z)$ . To this end, the natural assumption to make about  $\Gamma$  is that, for  $z \in U$ ,  $\sum_{T \in \Gamma} |T'(z)| < \infty$ . Since

$$(1.1) \quad (1 - |z|^2) \cdot |T'(z)| = 1 - |Tz|^2$$

the preceding inequality is equivalent to:

$$(1.2) \quad \sum_{T \in \Gamma} (1 - |Tz|^2) < \infty.$$

We shall therefore assume, unless otherwise stated, that *the group  $\Gamma$  is of convergence type, i.e.* that (1.2) is valid for some, and hence all,  $z$  in  $U$ . It is well-known (Tsuji [13], p. 522) that this is equivalent to the condition that the Riemann surface  $U/\Gamma$  is hyperbolic.

Let us now briefly indicate the contents of the paper. In §2 we introduce the basic definitions. The next section contains one of our basic results (Theorem 1): for an analytic function  $f$  in the Hardy class  $H^1(U)$ , the theta series of dimension  $-2$  exists. As an important application of this, we show (Th. 2) in Section 4 that the periods  $C(T)$  of a square integrable automorphic form of dimension  $-2$  satisfy the following condition:

$$\sum_{T \in \Gamma} |C(T)| \cdot (1 - |Tz|^2) < \infty \quad \text{for all } z \in U.$$

Section 5 contains some boundedness theorems for Poincaré series of dimension  $-2$  analogous to those known for dimensions  $< -2$  (cf: Ahlfors [1], Earle [6],

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