

The Condition for Uniqueness of the Dirichlet Problem for Hyperbolic Equations in Cylindrical Domains

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In this note we obtain the necessary and sufficient condition for uniqueness of solution of the Dirichlet problem for second order linear hyperbolic partial differential equations with variable coefficients in finite $(n + 1)$ -dimensional cylindrical domains with generators parallel to the t -axis. It is well known that the Dirichlet problem for hyperbolic equations is not in general a well posed problem. For example, the solution of the two-dimensional wave equation $u_{xx} - u_{tt} = 0$, in a rectangle with sides forming 45-degree angles with the coordinate axes, is completely determined by its values on only two adjacent sides of the rectangle. On the other hand, if the rectangle has sides parallel to the coordinate axes, it has been shown by Bourgin and Duffin [1] that uniqueness of solution of the Dirichlet problem holds if and only if the ratio of the sides of the rectangle is an irrational number. Related studies of the problem for the two-dimensional wave equation, concerning questions of uniqueness, existence and continuous dependence of the solution on the boundary data, have also been carried out by John [2] and Fox and Pucci [3].

The result of Bourgin and Duffin was extended by the present authors [4] to the case of the $(n + 1)$ -dimensional wave equation in rectangular domains bounded by hyperplanes normal to the coordinate axes. We use the same method of proof for the more general problem of this paper. The condition for uniqueness is expressed in a more meaningful context and, naturally, it contains all previous results as special cases.

Let D be a bounded domain in n -dimensional space R_n and let $x = (x_1, \dots, x_n)$