

Lipschitz Classes and Fourier Coefficients

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1. Introduction.

1.1. Let f be an integrable function with period 2π and be even or odd and let

$$f(x) \sim \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{or} \quad f(x) \sim \sum_{n=1}^{\infty} a_n \sin nx.$$

We suppose

$$0 < \alpha \leq 1, \quad p \geq 1 \quad \text{and} \quad r \geq 1.$$

The Lipschitz class $\text{Lip } \alpha$ is the space of all functions f satisfying the condition

$$(1.1) \quad \|f\|_{\alpha} \equiv \sup_{t>0, x} \left(\frac{|f(x+t) - f(x)|}{t^{\alpha}} \right) < \infty.$$

The space Λ (Zygmund class) is the space of all functions f satisfying the condition

$$\|f\|_{\Lambda}^* \equiv \sup_{t>0, x} \left(\frac{|f(x+t) - 2f(x) + f(x-t)|}{t} \right) < \infty.$$

The Hardy-Littlewood class $\text{Lip } (\alpha, p)$ ($p < \infty$) is the space of all functions f satisfying the condition

$$(1.2) \quad \|f\|_{\alpha, p} \equiv \sup_{t>0} \left(\int_0^{2\pi} \left| \frac{f(x+t) - f(x)}{t^{\alpha}} \right|^p dx \right)^{1/p} < \infty.$$

Recently, the space $\Lambda(\alpha, p, r)$ was defined by M. H. Taibleson and R. Askey as the set of all functions f satisfying the condition

$$(1.3) \quad \|f\|_{\alpha, p, r} \equiv \left(\int_0^1 \left(\int_0^{2\pi} \left| \frac{f(x+t) - 2f(x) + f(x-t)}{t^{\alpha}} \right|^p dx \right)^{r/p} \frac{dt}{t} \right)^{1/r} < \infty.$$

Then we can easily see that

$$\Lambda(\alpha, \infty, \infty) = \text{Lip } \alpha \quad \text{for } 0 < \alpha < 1,$$

$$\Lambda(1, \infty, \infty) = \Lambda,$$

$$\Lambda(\alpha, p, \infty) = \text{Lip } (\alpha, p) \quad \text{for } 0 < \alpha \leq 1 \quad \text{and} \quad 1 \leq p < \infty.$$