

# *Spline-Blended Surface Interpolation Through Curve Networks*

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*Communicated by* GARRETT BIRKHOFF

**Abstract.** The purpose of this paper is to describe and analyze the new family of methods for bivariate interpolation first reported in [12]. Although univariate spline functions play a major role in their construction, these bivariate interpolation functions are not tensor product splines. Indeed, the function space associated with this family of methods is infinite-dimensional. Whereas tensor product ("cross product") methods of interpolation such as de Boor's bicubic splines [8] and their generalizations (*cf.* Lemma 4) interpolate to discrete *data* at mesh-points, the methods described herein (*cf.* Theorem 1) interpolate to arbitrary continuous *functions* given along mesh-lines. These new methods do, however, include the familiar product methods as a special case. In [5], S. A. Coons described a class of bivariate interpolation schemes which have come to be known as "(polynomial) blending-function" methods (*cf.* also [4]). Since the methods described herein are extensions of the Coons-type methods in the same sense as two-endpoint spline interpolation (*cf.* Lemma 1) is an extension of two-endpoint (Hermite) polynomial interpolation, we shall refer to this new family of methods as *spline-blended bivariate interpolation*. Although the principal emphasis throughout the paper is on interpolation, in Section 5 we outline some related results concerned with bivariate approximation in the  $l_2$  norm.

**1. Introduction.** Let  $Z = U(x, y)$  be any continuous surface defined over the rectangular domain  $\mathcal{R} = [a, b] \times [c, d]$  in the  $(x, y)$ -plane. Let  $\pi : a = x_1 < x_2 < \cdots < x_N = b$  and  $\pi' : c = y_1 < y_2 < \cdots < y_{N'} = d$  be partitions of the intervals  $I = [a, b]$  and  $I' = [c, d]$ , and consider the *plane sections* of  $U$  cut by the vertical planes  $x = x_i$  and  $y = y_j$

$$(1) \quad \begin{aligned} g_i(y) &= U(x_i, y) && (i \in \overline{1, N}) \\ h_j(x) &= U(x, y_j) && (j \in \overline{1, N'}); \end{aligned}$$