

# On the Finite Generation of $\Omega_*^U(X)$

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Our objective in this note is to establish:

**Theorem.** *If  $X$  is a finite polyhedron then  $\Omega_*^U(X)$  is a finitely generated  $\Omega_*^U$ -module.*

Here of course  $\Omega_*^U(X)$  denotes the almost weakly complex bordism of the space  $X$  [1] [5]. Recall that [5]  $\Omega_*^U = \mathbf{Z}[x_1, \dots, x_n, \dots]$  where  $\deg x_n = 2n$ . Therefore  $\Omega_*^U$  is not a Noetherian ring. An adequate substitute in this case for the Noetherian condition is the property of coherence. This is discussed in the first section and applied in the second section to deduce the above result.

## 1. Coherent rings and modules.

**Notation.**  $\Omega$  always denotes a ring with 1. Module always means left unital  $\Omega$ -module.

**Definition.** A presentation of an  $\Omega$ -module  $M$  is an exact sequence of  $\Omega$ -modules

$$0 \rightarrow R \rightarrow F \rightarrow M \rightarrow 0$$

where  $F$  is a free  $\Omega$ -module. A presentation of  $M$  is called *finite* if  $F$  and  $R$  are finitely generated  $\Omega$ -modules.

An  $\Omega$ -module  $M$  is *coherent* iff  $M$  and all of its finitely generated submodules are finitely presentable.

Note that a coherent  $\Omega$ -module is always finitely generated.

**Definition.** A ring  $\Omega$  is *coherent* iff  $\Omega$  is coherent as an  $\Omega$ -module.

Note that any Noetherian ring is coherent. The converse is false as is shown by:

**Proposition 1.1.** *If  $\mathbf{k}$  is a Noetherian ring then  $\Omega = \mathbf{k}[x_1, \dots, x_n, \dots]$  is a coherent ring.*

**Proof.** When viewed as a module over itself the  $\Omega$ -submodules of  $\Omega$  are just the ideals in  $\Omega$ . Let  $I \subset \Omega$  be a finitely generated ideal with generators  $f_1, \dots, f_t$ .