

Irreducible Almost Periodic Markov Operators

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1. Introduction. Let S be a compact Hausdorff space and $C(S)$ the space of continuous functions on S . Let T be a non-negative linear operator on $C(S)$ with $T1 = 1$. We will call T a *Markov operator*. The dual $C^*(S)$ of $C(S)$ is the space of Radon measures on S and $K(S)$ is the w^* compact convex subset of Radon probabilities ($\mu \in C^*(S)$ with $\mu(1) = 1 = \|\mu\|$). It is easily seen that T^* takes $K(S)$ into itself and therefore there will always exist invariant probabilities. The weak* continuous map of S into $K(S)$ given by $x \rightarrow T^*\xi_x = P(x, \cdot)$ will be called the *transition function* (ξ_x is the unit mass at x measure). The iterates of T are again Markov operators so that the n^{th} order transition function $P_n(x, \cdot)$ is defined as the transition function of T^n . We say that T is *irreducible* if for each point x in S and each open set U in S there is an integer n so that $P_n(x, U) > 0$. If for each f in $C(S)$ every subsequence of the sequence of iterates $\{T^n f: n \geq 1\}$ has a norm convergent subsequence we call T *almost periodic*. In the case that each subsequence has a subsequence which is convergent in the weak topology (so that $T^n f$ converges pointwise for n in E where E is the subset of the integers determining the subsequence) we call T *weakly almost periodic*. If T is irreducible and (weakly) almost periodic then it is known that the de Leeuw–Glicksberg theory of almost periodic compactifications [8] (with an assist from [13] in the weak case) can be applied to obtain a decomposition of $C(S) = C_p \oplus C_0$ which reduces T . C_p is the closed linear span of the complex valued eigenfunctions with unimodular eigenvalues and C_0 is the space of all f in $C(S)$ with $\|T^n f\| \rightarrow 0$ ($T^n f \rightarrow 0$ pointwise in the weak case). In the interest of completeness we give in the third section a self-contained development of this decomposition which avoids abstract algebraic analysis. The second section is devoted to certain topological and spectral properties which depend on irreducibility alone for the most part. A *deterministic Markov operator* is generated by a continuous point map $\varphi: S \rightarrow S$ by $Tf(x) = f \circ \varphi(x)$. A Markov operator is deterministic iff $T(fg) = (Tf)(Tg)$ because the only probabilities which are multiplicative func-

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