

A Homotopy Group of Algebraic Arcs Found by Smoothed Approximations

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Introduction. The paper develops the notion of homotopy group on a real algebraic variety V where the paths and homotopy satisfy certain analytic conditions.

Definition 1. $f \in T(V, P) \Leftrightarrow f: I \rightarrow V$ is continuous and $\exists t_i$, *partition numbers* of f where $0 = t_0 < t_1 < \dots < t_n = 1$ such that

- 1) $f|_{(t_i, t_{i+1})} \rightarrow V$ is a non-singular algebraic path (open) or
- 2) $f|_{[t_i, t_{i+1}]} \rightarrow \{P\}$ where $P \in V$
- 3) $f(t) = P$ if $t = t_i$ for some i
- 4) $f|_{\subset \mathfrak{S}(V)}$, the singular locus of V (unless $f = P$).

$f \in T(V, P)$ is *non-reversing* if there are never t, t', t'' such that

$f|_{[t, t']} \rightarrow V$ is the same as path $f|_{[t', t'']}$ except traversed in the opposite direction.

Definition 2. Let $H: I \times I \rightarrow V$ be analytic, $H(0 \times I), H(I \times I)$ be *edges* of \overline{imH} as in case 1 or 2 of Definition 1, nonsingular (except possibly at P) on \overline{imH} (the algebraic closure of imH , imH a 2-dimensional subset of a 2-dimensional algebraic variety) in case 1. If \overline{imH} contains no irreducible arcs α singular on \overline{imH} containing a point R singular on $\bar{\alpha}$ (a 0/1 dimensional singularity), then \overline{imH} will be called an algebraic square.

Definition 3. A P -*chained algebraic squares* S with *partition numbers* $\{t_i\}$ where $0 = t_0 < t_1 < t_2 < \dots < t_n = 1$ and *special partition numbers* $\{t_{i_k}\}$ where $t_{i_k} \in \{t_i\}$ is a square defined by a continuous map H

$$H: I \times I \rightarrow V$$

such that

a) If t_i is not a special partition number, then

- 1) α) There are numbers $0 = s_0 < \dots < s_n = 1$ such that

$$H|_{[s_i, s_{i+1}] \times [t_i, t_{i+1}]} \rightarrow V$$