

# *On the Existence of Optimal Random Controls*

H. BECKER & V. MANDREKAR

*Communicated by* MURRAY ROSENBLATT

Dedicated to Professor L. C. Young on his 63rd birthday

**1. Introduction.** The study of existence of an optimal control has been the subject of several papers (see [2], [3]). Most of the results in this direction are fragmentary. Recently, E. J. McShane in his review paper [5] and L. C. Young in his notes [10] have shown a new approach to the deterministic problem by using what are termed as generalized controls. In this case the existence problem is treated in a natural way. Our purpose here is to investigate the corresponding random control problem. We introduce a class  $\mathcal{G}$  of generalized random controls and show that in this class there exists an optimal control.

The control problem under consideration is represented by the following stochastic integral equation “with memory” subject to some past condition (*cf.* (4.2))

$$(1.1) \quad X(t) = X(0) + \int_0^t \int_{\mathfrak{U}} \alpha(\tau, X(s), u; s \leq \tau) \mu_\tau(du) d\tau \\ + \int_0^t \beta(\tau, X(s), B(s); s \leq \tau) B(d\tau),$$

where (i) the state  $X(t)$  of the system is a Hilbert-space valued random variable for each time point  $t$ , (ii)  $\mu_\tau(du)$  are unit measures on the control space  $\mathfrak{U}$  with probability one and (iii) the first integral is defined as the Bochner integral and the second is the Hilbert-space valued Ito-integral with respect to Brownian motion  $B$ . (R. E. Mortensen [6]).

The equation (1.1) is a generalization of the one due to W. Fleming and M. Nisio ([2] (1.1)) not only in the integrand but in their situation the random measure  $\mu_\tau$  of the first integral is a measure concentrated at a point in the control region  $\mathfrak{U}$  for each time point  $\tau$ . The problem is to find a control  $W = \mu_\tau(du)d\tau$

Sponsored by the Mathematics Research Center, United States Army, Madison, Wisconsin, under Contract No. DA-31-124-ARO-D-462.