The Initial Value Problem for the Linearized Equations of Water Waves, II

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1. Introduction. Let $x = (x', y) = (x_1, \dots, x_{n-1}, y)$ denote a variable point in \mathbb{R}^n . Let G be a domain in \mathbb{R}^n whose boundary consists of two parts, Γ and Γ' . Γ is an (n-1)-dimensional subdomain of the plane y = 0 and is conceived of as the free surface of a fluid that fills G. Γ' is a fixed boundary through which no fluid can flow. Let

$$\triangle = \sum_{1}^{n-1} \frac{\partial^{2}}{\partial x_{i}^{2}} + \frac{\partial^{2}}{\partial y^{2}},$$

let ν denote the outward normal to Γ' , and let subscripts denote differentiation. We seek a function u(x, t) defined on $\bar{G} \times [0, \infty)$ satisfying the following equations:

$$(1.1) \Delta u = 0 for x e G, t \ge 0,$$

(1.2)
$$u_{\nu} = 0 \quad \text{for} \quad x \in \Gamma', \qquad t \ge 0,$$

(1.3)
$$u_y + u_{tt} = 0 \quad \text{for} \quad x \in \Gamma, \qquad t \ge 0,$$

(1.4)
$$u = u^0$$
, $u_t = u_t^0$ for $x \in \Gamma$ when $t = 0$.

Here, u^0 and u_t^0 are given functions, the initial data of the problem. t is time, which makes its sole appearance as a parameter with respect to which the unknown function u is differentiated. (See [1] for a derivation of (1.1-4) and a discussion of their physical significance.)

In a paper [1] with the same title as the present one, we solved the system (1.1-4) for very general domains G, the so-called *normal* domains. It was shown that conditions (1.1), (1.2), and (1.4) are satisfied classically but, in general, that (1.3) is satisfied only in a weak sense. With a bit of smoothness on the initial data, we showed that (1.3) is satisfied for almost all x' in Γ , but that was the best we could do.

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