

Weakly Wandering Sets and Invariant Measures for a Group of Transformations

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§1. Introduction. Let (S, \mathfrak{F}, m) be a finite or σ -finite measure space, and let $\mathcal{G} = \{g\}$ be a semigroup of measurable, non-singular, and not necessarily invertible transformations defined on (S, \mathfrak{F}, m) . The problem of finding necessary and sufficient conditions for the existence of a finite invariant measure μ equivalent with m has been discussed by many authors for the case when \mathcal{G} is a cyclic group, and a number of interesting conditions have been found, see [2], [5], [6], [8], [10] and [11]. While most of these conditions obtained for the special case do not have a meaningful extension to a more general class of transformations, some of them can be extended in an obvious way to an arbitrary semigroup of transformations. We can, for example, extend the notion of a weakly wandering set, defined first for the case of a cyclic group by Hajian and Kakutani in [10]. In [10] it was shown that there exists a finite invariant measure μ equivalent with m for a cyclic group \mathcal{G} if and only if

(V) There does not exist any weakly wandering set of positive measure under \mathcal{G} .

Recently, [1], [4], [9], [12], [13], [14], [15], [16], [17], [18], and [19] have extended the result of Hajian and Kakutani mentioned above to show that Condition (V) is necessary and sufficient for the existence of a finite invariant measure μ for more general classes of transformations. So far the most general result was the one obtained by Natarajan, [13] and [14], which stated that Condition (V) remains to be necessary and sufficient if \mathcal{G} is a left-amenable semigroup of non-singular and measurable transformations.

In this paper we treat the existence of an invariant and equivalent measure as a problem of finding a strictly positive fixed point under a semigroup of positive contraction operators defined on an appropriate Banach space. Via this approach we succeed in obtaining a more general result than those mentioned above; namely, we prove that Condition (V) is in fact necessary and sufficient for the existence of a finite invariant measure μ equivalent with m

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