

Removable Sets for Quasiconformal Mappings

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It is well known that an isolated boundary point is a removable singularity for quasiconformal mappings. This means that if D is a domain in the euclidean n -space R^n and if $b \in D$, then every quasiconformal mapping f of $D - \{b\}$ can be extended to a quasiconformal mapping of D . For $n = 2$ the proof is given in [3, p. 43], and the general case can be treated by a similar method. The purpose of this note is to show that this result remains true if the point b is replaced by a set of $(n - 1)$ -dimensional measure zero. For $n = 2$ this is well known, see [3, p. 216]. However, the 2-dimensional proof cannot be directly generalized to higher dimensions. Our proof is based on some results of Callender and Rešetnyak.

We first introduce some notations and terminology. Suppose that D is a domain in R^n . A mapping $f : D \rightarrow R^n$ is said to be ACL (absolutely continuous on lines) if for every $1 \leq i \leq n$ and for almost every line L , parallel to the x_i -axis, f is absolutely continuous on every compact interval of $L \cap D$. Note that f is not required to be continuous. An ACL-mapping f is said to be ACL^p , $p \geq 1$, if (1) f is locally integrable, (2) the partial derivatives $\partial_i f(x)$ exist a.e. and (3) these partial derivatives are locally L^p -integrable. In the literature, the class of ACL^p -mappings is often denoted by W_p^1 or by H_p^1 . If f is ACL^p , we denote by $f'(x)$ the linear mapping of R^n into R^n , defined by $f'(x)e_x = \partial_x f(x)$, where e_1, \dots, e_n are the coordinate unit vectors. Furthermore, we use the notations

$$|f'(x)| = \max_{|h|=1} |f'(x)h|, \quad J(x, f) = \det f'(x).$$

Thus $f'(x)$ and $J(x, f)$ are defined a.e. in D .

Following Rešetnyak [5], we say that an ACL^n -mapping $f : D \rightarrow R^n$ is of bounded distortion if there is a finite constant K such that

$$(1) \quad |f'(x)|^n \leq KJ(x, f)$$

a.e. in D . If, in addition, f is a homeomorphism onto a domain $D' \subset R^n$, it is called quasiconformal. A quasiconformal mapping satisfies also a condition

$$(2) \quad J(x, f) \leq K(\min_{|h|=1} |f'(x)h|)^n$$