## A New Type of Multidimensional Singular Linear Integral Equation\*

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I. Introduction. In the theory of radiative transfer through a plane-parallel atmosphere, the optical properties of which vary with optical depth, one has to solve the following system of nonlinear singular integral equations [3]:

$$(1a) X^*(\alpha, \beta; \mu) = 1 + \int_0^1 \left[ \int_{\alpha}^{\beta} X^*(\alpha, \beta'; \mu) X^*(\alpha, \beta'; \mu') \Omega(\beta') \right] \cdot \exp\left\{ -(\beta - \beta') \left( \frac{1}{\mu} + \frac{1}{\mu'} \right) \right\} d\beta' \right] \psi(\mu') \frac{d\mu'}{\mu'},$$

$$(1b) Y^*(\alpha, \beta; \mu) = \exp\left\{ \frac{-(\beta - \alpha)}{\mu} \right\} + \int_0^1 \left[ \int_{\alpha}^{\beta} Y^*(\alpha, \beta'; \mu) X^*(\alpha, \beta'; \mu') \Omega(\beta') \right] \cdot \exp\left\{ \frac{-(\beta - \beta')}{\mu'} \right\} d\beta' \right] \psi(\mu') \frac{d\mu'}{\mu'},$$

$$(1c) X(\alpha, \beta; \mu) = 1 + \int_0^1 \left[ \int_{\alpha}^{\beta} Y^*(\alpha, \beta'; \mu) Y^*(\alpha, \beta'; \mu') \Omega(\beta') d\beta' \right] \psi(\mu') \frac{d\mu'}{\mu'},$$

$$(1d) Y(\alpha, \beta; \mu) = \exp\left\{ -(\beta - \alpha) \right\} + \int_0^1 \left[ \int_{\alpha}^{\beta} Y^*(\alpha, \beta'; \mu) Y^*(\alpha, \beta'; \mu') \Omega(\beta') d\beta' \right] \psi(\beta') \Omega(\beta')$$

(1d) 
$$Y(\alpha, \beta; \mu) = \exp\left\{\frac{-(\beta - \alpha)}{\mu}\right\} + \int_0^1 \left[\int_{\alpha}^{\beta} X^*(\alpha, \beta'; \mu) Y^*(\alpha, \beta'; \mu') \Omega(\beta') \cdot \exp\left\{\frac{-(\beta - \beta')}{\mu}\right\} d\beta'\right] \psi(\mu') \frac{d\mu'}{\mu'},$$

where the positive constants  $\alpha$  and  $\beta$  are the optical depths at the boundaries of the atmosphere,  $\mu$  and  $\mu'$  are the cosines of the angles of emergence and incidence, respectively, measured from the outward normal,  $\Omega(\beta')$  is the variable

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