

# On the Topological Entropy of Solenoids<sup>1</sup>

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Communicated by G.-C. ROTA

In this note we prove a theorem which relates the entropy of a shift on an inverse limit space to the entropy of the map used in the construction of the inverse limit which is analogous to a theorem for measure theoretic entropy [3]. Using this theorem we then compute the topological entropy of the shift on a  $p$ -adic solenoid.

**I. The main theorem.** In the following an open cover  $\mathfrak{U}$  of a topological space  $X$  will be called minimal if no sub cover of  $\mathfrak{U}$  covers  $X$  and a sub cover  $\mathfrak{U}'$  of an open cover  $\mathfrak{U}$  of  $X$  will be called a minimal sub cover if  $\mathfrak{U}'$  is a minimal cover of  $X$  and no sub cover of  $\mathfrak{U}$  contains fewer sets than  $\mathfrak{U}'$ .

Throughout,  $X$  will denote a compact topological space and  $\varphi$  a continuous map of  $X$  onto  $X$ . If  $\mathfrak{U}$  is an open cover of  $X$ ,  $N(\mathfrak{U})$  will denote the number of sets in a minimal sub cover of  $\mathfrak{U}$ . We will denote the topological entropy of  $\varphi$  with respect to  $\mathfrak{U}$  by  $h(\varphi, \mathfrak{U})$ , and  $h(\varphi)$  will denote the topological entropy of  $\varphi$ . For the definitions of these objects and their properties one may consult [1].

By  $X_\infty$  we shall denote the subspace of the product space of countably many copies of  $X$  with the product topology which consists of all sequences  $(x_1, x_2, \dots)$  such that  $\varphi(x_n) = x_{n-1}$  for  $n \geq 2$ . The projection maps of  $X_\infty$  onto  $X$  will be denoted by  $\pi_k$  for  $k \geq 1$  and  $\varphi_*$  will denote the shift on  $X_\infty$ , i.e.  $\pi_k \circ \varphi_* = \pi_{k+1}$ . It is known [2, Chapter VIII] that  $X_\infty$  is a compact topological space,  $\varphi_*$  is a homeomorphism of  $X_\infty$  and  $\pi_k$  are continuous surjections. Further, note that

$$(A) \quad \varphi \circ \pi_k = \pi_k \circ \varphi_*^{-1}$$

and

$$(B) \quad \varphi \circ \pi_{k+1} = \pi_k$$

for all  $k \geq 1$ .

**1. Lemma.** Let  $\{U_i : i \in I\}$  be any collection of subsets of  $X$ , and  $k$  a positive integer. Then  $X = \cup_i U_i$  if and only if  $X_\infty = \cup_I \pi_k^{-1}(U_i)$ .

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<sup>1</sup> Research supported in part by ARO Grant 662

<sup>2</sup> National Science Foundation Fellow