

Representation of Semi-Groups of Nonlinear Nonexpansive Transformations in Banach Spaces

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Let S be a weakly complete Banach space with conjugate space S^* separable. A semi-group of nonlinear nonexpansive transformations on S is a function T from $[0, \infty)$ to the set of transformations on S such that

- (1) $T(x + y) = T(x)T(y)$ for all $x, y \geq 0$,
- (2) $T(x)$ is nonexpansive for each $x \geq 0$, i.e., if $p, q \in S$ then $\|T(x)p - T(x)q\| \leq \|p - q\|$,
- (3) if $p \in S$ and $g_p(x) = T(x)p$ for each $x \geq 0$, then g_p is continuous and $g_p(0) = p$.

If there is a point q in S such that $g_q^+(0)$ exists, then the infinitesimal generator of T is defined to be the function A such that $Ap = g_p^+(0)$ for all p in S for which $g_p^+(0)$ exists.

If for each $x \geq 0$ it is required that $T(x)$ is a linear transformation on S then T is a semi-group of linear nonexpansive transformations on S and it can be shown (see [4] or [12]) that the infinitesimal generator A of T is densely defined, for each $\epsilon > 0$ $(I - \epsilon A)^{-1}$ exists and has domain S , and T may be represented as follows:

$$T(\epsilon)p = \lim_{n \rightarrow \infty} (I - (\epsilon/n)A)^{-n}p \quad \text{for } p \in S, \quad \epsilon \geq 0.$$

If T is a semi-group of nonlinear nonexpansive transformations on S , J. W. Neuberger [9] has shown the following theorem:

If $x > 0$ and $A_x = (1/x)[T(x) - I]$, then for each $\epsilon \geq 0$ $(I - \epsilon A_x)^{-1}$ exists, has domain S , and is nonexpansive. Further, if there is a dense subset D of S

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