

# On Certain Fractional Area Integrals

C. SEGOVIA & R. L. WHEEDEN\*

*Communicated by* MAURICE HEINS

**§0. Introduction.** Let  $F(x, t)$  be the gradient of a function  $U(x, t)$  which is defined and harmonic in  $E_{n+1}^+ = \{(x, t) : x = (x_1, \dots, x_n) \in E_n, t > 0\}$ —i.e.,

$$F(x, t) = \left( \frac{\partial U}{\partial x_1}, \dots, \frac{\partial U}{\partial x_n}, \frac{\partial U}{\partial t} \right)(x, t).$$

Following [9], we say  $F$  belongs to the Hardy class  $H^p$  if there exists a constant  $K > 0$  such that

$$\int_{E_n} |F(x, t)|^p dx \leq K^p$$

for every  $t > 0$ . We will always assume that  $p > (n-1)/n$ . Denoting  $F'(x, t) = (\partial/\partial t)F(x, t)$ , we set

$$g^{(F)}(x) = \left( \int_0^\infty t |F'(x, t)|^2 dt \right)^{1/2},$$

$$S(F)(x) = S(F)(x; \epsilon) = \left( \iint_{|x-y| < \epsilon t} |F'(y, t)|^2 \frac{dy dt}{t^{n-1}} \right)^{1/2}$$

and

$$g^*(F)(x) = g^*(F)(x; \lambda) = \left( \iint_{t>0} \frac{t^\lambda}{(|x-y|+t)^\lambda} |F'(y, t)|^2 \frac{dy dt}{t^{n-1}} \right)^{1/2}$$

for  $\lambda > 1$ .

These well known functions of Littlewood–Paley, Lusin, and Littlewood–Paley–Zygmund respectively have been studied extensively and share many important properties with the vector  $F$ . They satisfy the inequalities

$$g(F)(x) \leq cS(F)(x; \epsilon) \leq c_{\epsilon, \lambda} g^*(F)(x; \lambda)$$

(see [4], p. 447).

Our first goal is to generalize these integrals as follows. Given  $\alpha > 0$ , let

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\* This research was partially supported by the U. S. Air Force Office of Scientific Research 68-1467 for the first author and by NSF GP 8556 for the second author.