

# *Linear Functionals and the Cauchy-Kowalewski Theorem*

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**1. Introduction.** The purpose of this note is to show how the classical Cauchy-Kowalewski theorem, the Cauchy integral formula, and the calculus of residues in several complex variables can all three be combined to yield, in a constructive fashion, formulas which represent the solution of the linear non-characteristic Cauchy problem as a linear functional of its Cauchy data. The formal construction is based on an extremely simple idea. Its rigorous justification relies on the method of majorants and uses essentially the same bounds that occur in the proof of the Cauchy-Kowalewski theorem. In many special cases where it can be calculated explicitly the general representation we obtain here reduces to well known classical formulas.

Recall that the non-characteristic Cauchy problem for an arbitrary linear system can be reduced to a Cauchy problem for an  $N \times N$  linear first-order system in  $n + 1$  variables of the form

$$(1) \quad \frac{\partial u}{\partial t} - \sum_{k=1}^n a^k(t, x) \frac{\partial u}{\partial x_k} - b(t, x)u = g(t, x)$$

$$u(0, x) = f(x).$$

Here everything is complex:  $t \in \mathbf{C}$ ,  $x = (x_1, x_2, \dots, x_n) \in \mathbf{C}^n$ , the  $a^k, b$  are  $N \times N$  matrices, and the  $u, g, f$  are  $N$ -tuples. We assume that the  $a^k, b, f, g$  are holomorphic functions of all of their arguments in some polydisc. Then according to the Cauchy-Kowalewski theorem there is some smaller polydisc in which there exists a unique holomorphic solution  $u$  of (1).

We claim that  $u$  has a representation of the form

$$(2) \quad u(t, x) = \text{Res} \int_{\partial(0)} K(t, x; 0, \xi) f(\xi) d\xi$$

$$+ \int_0^t \text{Res} \int_{\partial(\tau)} K(t, x; \tau, \xi) g(\tau, \xi) d\xi d\tau.$$

The precise meaning of (2) will be explained below. When  $g \equiv 0$  equation (2)