

On Existence, Uniqueness and Convergence of Successive Approximations for Hyperbolic Partial Differential Equations

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In [1] the author generalized the Krasnoselskii–Krein criteria for ordinary differential equations, to hyperbolic partial differential equations in normal form. Also, examples were provided to show optimality of the conditions imposed. Following the same line of thought, an extension of Kooi's condition [2] will be given. Since similar techniques and a similar approach will be used, we don't provide the proofs but only state the final results.

Definition. Let $f(x, y, \dots, p_{ii}, \dots)$ be defined in

$$D = \begin{cases} (x, y) \in R \equiv [0, a] \times [0, b] \\ -\infty < p_{ii} < +\infty \\ 0 \leq i \leq r, \quad 0 \leq j \leq s, \quad r + s = n, \quad i + j \leq r + s - 1 \end{cases}$$

where it is also continuous. We say that f satisfies in D a *generalized Kooi condition of order* (r, s) if, for $(x, y, \dots, p_{ii}, \dots), (x, y, \dots, \bar{p}_{ii}, \dots) \in D$:

$$\begin{aligned} \text{A)} \quad & x^\alpha y^\beta |f(x, y, \dots, p_{ii}, \dots) - f(x, y, \dots, \bar{p}_{ii}, \dots)| \\ & \leq \sum_{(i,j)} K_{ij}(x, y) x^i y^j |p_{ii} - \bar{p}_{ii}|^\alpha \\ \text{B)} \quad & |f| \leq H x^\alpha y^\beta \end{aligned}$$

where:

$$\begin{aligned} \text{a)} \quad & \{K_{ij}(x, y)\} \subset C(R), \quad K_{ij}(x, y) \geq 0 \quad \forall (i, j) \\ & K = [\max_R (\max_{(i,j)} K_{ij}(x, y))] > 0 \end{aligned}$$

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