

On Weinstein's Intermediate Problems for Integral Equations with Difference Kernels

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1. Introduction. The Ritz-Rayleigh procedure for estimating the eigenvalues of a compact, self-adjoint operator A leads to a finite-dimensional problem having, in general, some positive and negative eigenvalues which, with the usual ordering, provide lower bounds to the positive eigenvalues of A and upper bounds to the negative eigenvalues of A , respectively.

If A is definite, say positive, it seems natural to use Weinstein's method of intermediate problems (see [2], [4] for a detailed description) to obtain complementary bounds to the eigenvalues. The method relies on a base operator B , with known spectrum, whose eigenvalues are larger than those of A . If B is itself compact and self-adjoint, the eigenvalues of successive intermediate problems decrease monotonically to those of A . On the other hand, Fichera [3] has shown that a non-compact base operator B may fail to yield suitable intermediate problems. When A is a definite integral operator, Weinstein [10] has constructed, in a simple and elegant manner, a non-compact base operator suitable for the application of the method of intermediate problems. This base operator must be modified in an elementary way at each stage in the succession of intermediate problems.

The purpose of the present note is to construct a *compact* base operator whenever A is a compact, self-adjoint, integral operator generated by a difference kernel. The base operator remains fixed for the various intermediate problems. Moreover, the usual restriction of A being definite is unnecessary. The procedure yields bounds complementary to the Ritz-Rayleigh bounds; thus one obtains upper bounds to the positive eigenvalues and lower bounds to the negative eigenvalues. The base operator is constructed by extending the kernel to a larger interval over which it has an appropriate periodicity. The spectrum of the base operator can be calculated explicitly by Fourier expansion. The same kind of periodic extension has been used by Rosenblatt [5] in discussing the asymptotic behavior of eigenvalues. An application to the estimation of functionals associated with inhomogeneous equations was given in [6].

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