

*The deRham Theorem for Function Spaces*¹

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1. Introduction. The classical theorem of deRham states that there is a degree preserving isomorphism from the real singular cohomology of a paracompact smooth (C^∞) manifold onto the cohomology of smooth forms via the integral.

In this note we prove that this theorem is valid for the group of diffeomorphisms of a compact smooth manifold and for the space of smooth maps from a compact smooth manifold into a paracompact smooth manifold with C^∞ -topologies. The essential difficulty is the definition of differential forms.

One approach to such a theorem is given by Eells in [1]. Such a method is not feasible in our case since the function spaces we are concerned with are locally homeomorphic to Montel spaces and hence cannot be normed.

Thus in order to avoid the theory of differentiation in locally convex topological vector spaces we have been inspired by an idea of J. W. Smith [5]. The point of departure here is a new form of differentiability.

To provide some continuity we briefly summarize [2] and [5]. Let \mathfrak{M} be the category of smooth manifolds and smooth maps. The smooth maps of U into V are denoted by $C^\infty(U, V)$ for $U, V \in \mathfrak{M}$; the continuous maps of X into Y are denoted by $C(X, Y)$ for X, Y topological spaces.

Let X be a topological space and \mathfrak{F} a subsheaf of $C(X, \mathbf{R})$. For any manifold U and $\alpha \in C(U, X)$, α is called differentiable (of class C^∞) iff $\alpha^*\mathfrak{F} \subset C^\infty(U, \mathbf{R})$ where α^* is the family of functions of the form $\alpha^*f = f \circ \alpha$, $f \in \mathfrak{F}$. In a similar manner for any $V \in \mathfrak{M}$, $\beta \in C(X, V)$ is called differentiable iff $\beta^*(C^\infty(V, \mathbf{R})) \subset \mathfrak{F}$. If \mathfrak{F} is to consist of differentiable functions one needs the following condition: If for all differentiable $\alpha \in C(U, X)$, $\beta \circ \alpha \in C^\infty(U, V)$ where $\beta \in C(X, V)$ then β is differentiable. A subsheaf $\mathfrak{F} \subset C(X, \mathbf{R})$ satisfying the condition is called an inductive differentiable structure or simply a differentiable structure.

Remarks. 1. If \mathfrak{W} is a functor which makes correspond to any manifold U , $\mathfrak{W}(U, X) \subset C(U, X)$, and to any $f \in C^\infty(U', U)$, $f_*: \mathfrak{W}(U, X) \rightarrow \mathfrak{W}(U', X)$ where $f_*\alpha = f\alpha$, $\alpha \in \mathfrak{W}(U, X)$ then there is a uniquely determined \mathfrak{F} such that

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