

*A Class of Wave Diffraction Problems Involving Geometrically-Induced Singularities*¹

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Abstract. In this paper, a general method is presented for solving two-dimensional diffraction problems for cases where the region under consideration is interrupted by a line of discontinuity. The same basic method of solution also solves the problem of a penny-shaped surface of discontinuity.

For systems governed by a set of n wave potentials satisfying Helmholtz equations, use is made of suitable integral transforms and each problem is reduced to the solution of a series of coupled Fredholm integral equations. The mathematical analysis covers a class of problems in the propagation of elastic, acoustic and electromagnetic waves. A number of examples from the field of elastodynamics are given including those of longitudinal shear and flexural waves incident on a slit of finite width. Special attention is directed to obtaining the details of the singular character of the near-field solution which has been known to control the instability behavior of geometrically-induced singularities.

1. Introduction. The problem of the scattering of waves (elastic, acoustic, electromagnetic, etc.) about obstacles has long been of great interest to both physicists and mathematicians because of its practical importance and the considerable mathematical difficulty encountered. The great majority of work in this field, however, has been highly specialized in the sense that it has been largely restricted to geometrically simple obstacles whose boundaries conform to coordinate curves or surfaces [1, 2]. In such instances, solutions may be obtained using the classical separation of variables method.

An important class of problems that has received relatively little attention is the diffraction of waves by lines or surfaces of discontinuity; here, the method of separation of variables fails to apply. In particular, knowledge of the detailed structure of solutions in the vicinity of the discontinuities is lacking. In elastodynamics [3], the principal interest is in the near-field solution from which the

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