

Steady Plane Magnetohydrodynamic Flow

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1. Introduction. A number of years ago, Martin [1] developed a theory of plane flow in which the pressure and the stream function were utilized as new independent variables replacing the usual spatial variables. In this theory, the solution of problems may be made equivalent to the problem of integrating a Monge–Ampère partial differential equation.

Later, Martin [2] showed that the determination of the plane steady flow of a fluid could be reduced to finding two functions, the Bernoulli function and the direction function, which together satisfy a quasi-linear partial differential equation. This equation could be used to determine one of these functions provided the other was prescribed. Applications of this method were given by Martin [3], [4] and Hansen and Martin [5].

In a recent paper [7], it was shown that Martin's Monge–Ampère formulation [1] could be extended to the class of perfectly conducting compressible fluids subjected to magnetic fields whose induction vector had two components zero. Further, the three possible cases could be studied virtually simultaneously; structurally, they led to the same differential equation; the only difference was in the definition of one of the independent variables, *i.e.*, the total pressure distribution. In the limit of vanishing magnetic field, the equations reduced exactly to the equations studied by Martin.

It is the purpose of this paper to extend Martin's other method [2] to the same special class of magnetohydrodynamic flows. In this way, it is possible to extend the idea of simple waves in rotational, iso-energetic flows to the magnetic case. As a very interesting by-product, it is found that the images of the characteristics in the hodograph plane form a fixed net of curves and do not depend on the solution under consideration, provided the flow is irrotational.

The analysis is based upon, and follows quite closely, the methods developed in [2]–[5]; generally speaking, the solutions of the present paper reduce exactly to the pertinent solutions in the cited references when the limit of vanishing magnetic field is taken.

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