

# *Inequalities Concerning the Modules of Curve Families*

WILLIAM R. DERRICK

*Communicated by G. SPRINGER*

**1. Introduction.** Let  $A$  be the topological image of an  $n$ -cube in  $n$ -dimensional Euclidean space and  $v, w$  be nonnegative Lebesgue-integrable real-valued functions defined on  $A$ . Then

$$(*) \quad \int_{\text{Int } A} vw \, dH^n \geq \left( \inf_{\Gamma} \int_c v \, dH^1 \right) \left( \inf_{\Sigma} \int_Q w \, dH^{n-1} \right),$$

where  $c$  is an arc joining, and  $Q$  is a set separating, the images of a fixed pair of opposite sides of the  $n$ -cube, and  $H^k$  is the  $k$ -dimensional Hausdorff measure in  $n$ -space. (\*) was conjectured, for  $n = 3$ , by R. J. Duffin [3] based on an analogous inequality for networks. Using (\*) one obtains, for arbitrary  $p > 1$ , the module inequality

$$M_p(\Gamma)M_p(\Sigma) \geq 1,$$

generalizing the identity

$$M_n(\Gamma)M_n(\Sigma) = 1,$$

due to Gehring [5] (for  $n = 3$ ) and Ziemer [6].

## 2. Preliminaries.

**2.1. Definitions.** Let  $E^n$  denote Euclidean  $n$ -space,  $\text{Bdry } A$ ,  $\text{Int } A$ , and  $\text{Diam } A$ , the *boundary*, *interior*, and *diameter* of a set  $A$  in  $E^n$  respectively,  $S_\epsilon(x) = \{y \text{ in } E^n : |x - y| < \epsilon\}$ ,  $I^n = [0, 1]^n$  the closed unit  $n$ -cube, and  $r_{xy}$  stands for the straight line segment from  $x$  to  $y$ .

A function  $f : E^n \rightarrow E^m$  is *approximately differentiable* at  $x$  if for some linear transformation  $D_{ap}f(x) : E^n \rightarrow E^m$  and for every  $r > 0$ , the point  $x$  is a point of density of the set

$$\{x : |f(x + h) - f(x) - D_{ap}f(x)(h)| \leq r |h|\}.$$