

*An Integral Identity Due to Ramanujan which Occurs in Neutron Transport Theory**

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1. **Introduction.** The large t asymptotic behavior of

$$(1) \quad N(t) = \int_0^\infty \frac{e^{-tx} dx}{x\{\pi^2 + (\log x)^2\}}$$

and a related family of functions is described incorrectly in the well-known Bateman Manuscript Project on higher transcendental functions. $N(t)$ is related to the special function $\nu(t)$ through an identity due to Ramanujan [1],

$$(2) \quad \nu(t) = \int_0^\infty \frac{t^x dx}{\Gamma(x+1)} = e^t - N(t).$$

The results stated in Bateman are incorrectly attributed to W. B. Ford [3]. They were apparently obtained by overlooking a restriction on a related theorem due to Ford. The correct asymptotic behavior is $N(t) = 1/\log t + O(1/(\log t)^2)$. Furthermore, the related functions $\nu(t, \alpha) - e^t$ differ radically from the description given in Bateman. All of these functions diverge when $\alpha > 0$ as $t \rightarrow \infty$ rather than approach zero as stated there.

Direct numerical evaluations of $N(t)$ have been performed which verify the results of our analysis. The incorrect description of $N(t)$ mentioned above gave physically unreasonable results for a problem in neutron transport theory which stimulated our original interest in this function and Ramanujan's identity.

The Laplace transform method is often used when investigating time-dependent problems of neutron transport theory. Typical difficulties which arise are the inversion of the related contour integrals and the determination of the asymptotic behavior of solutions for large times. The investigation of the time-dependence of a fast neutron field (with simultaneous fission regeneration and

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