

Compatibility Conditions of the Theory of Micromorphic Elastic Solids

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This paper is a contribution in honor of T. Y. THOMAS. It is an invited paper.

1. Introduction. Eringen and Suhubi [1, 2] gave a theory of microelasticity in which the local intrinsic motion of material elements is taken into account in the continuum behavior of elastic solids. This theory and its later extensions and specializations show that a large variety of physical phenomena, hitherto not encountered in the classical theories of elasticity and fluid dynamics, can be treated. For example, the effects of couple stress, distributed body couples, surface tensions, and distributed microcracks can be formulated mathematically and the underlying physical phenomena can be studied systematically. Other work related to this has appeared both in the United States and Europe. The aim of the present paper is not an assessment of existing literature, but to supply the compatibility conditions missing in these works. For a discussion of various theories, we refer the reader to [3] and [4].

2. Strain measures. According to the theory of micromorphic materials, (this terminology was introduced by Eringen [5] in his later work), the motion of a material volume element ΔV is described in terms of the "average motions" of large numbers of microvolume elements $\Delta V^{(\alpha)}$, ($\alpha = 1, 2, \dots$) contained in ΔV . The position vector of a material point in $\Delta V^{(\alpha)}$ is marked by $\mathbf{X}^{(\alpha)} = \mathbf{X} + \boldsymbol{\Xi}^{(\alpha)}$ where \mathbf{X} is the position vector of the center of mass P of ΔV and $\boldsymbol{\Xi}^{(\alpha)}$ is the relative position vector of a material point of $\Delta V^{(\alpha)}$ from the center of mass P , Fig. 2.1. The motion of the assemblage contained in ΔV is postulated to be

$$(2.1) \quad \mathbf{x}^{(\alpha)} = \mathbf{x}(\mathbf{X}, t) + \varkappa_{\kappa}(\mathbf{X}, t) \boldsymbol{\Xi}_{\kappa}^{(\alpha)},$$

where repeated indices indicate summation over the range (1, 2, 3). In (2.1) $\mathbf{x}(\mathbf{X}, t)$ is the spatial position p of the center of mass P at time t and the remaining

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