

# *A Novel Type of Matrix Multidimensional Singular Linear Integral Equation\**

A. L. FYMAT\*\* & K. D. ABHYANKAR†

*Communicated by I. N. SNEDDON*

**I. Introduction.** The theory of radiative transfer through a plane-parallel atmosphere of arbitrary stratification can be described locally or globally. In the local formulation, the radiative transfer equation and the auxiliary (multi-dimensional integral) equation satisfied by the source function constitute the relevant equations. The global (or state) formulation is based on the integro-differential equations for the reflection and transmission functions of the atmosphere. These formulations represent two equivalent descriptions of the same theory; accordingly, that formulation which is relatively more advantageous for the solution of the particular problem under consideration must be selected. When studying the transfer of electromagnetic radiation through inhomogeneous atmospheres, *viz.*, the absorbing and scattering properties of which vary with optical depth, we have found that the global formulation of the theory is to be preferred. This formulation makes use of the invariant character of the optical properties of the atmosphere with regard to the algebraic addition of an atmospheric layer of infinitesimally small optical thickness (Ambarzumian-Chandrasekhar invariance principles).

The eight integro-differential equations satisfied by the reflection and transmission functions represented by the  $4 \times 4$  matrices  $\mathbf{S}$  and  $\mathbf{T}$  in the case of an incident illumination from the top of the atmosphere, and by  $\mathbf{S}^*$  and  $\mathbf{T}^*$  in the case of an illumination from the bottom of the atmosphere, taking successively the parameters which characterize the atmospheric boundaries as independent variables, all exhibit the same kernel, namely the phase matrix  $\mathbf{P}(\mu, \phi; \mu', \phi')$ . Here,  $(\mu', \phi')$  denotes the directional parameters of the incident radiation, and  $(\mu, \phi)$ , the scattered radiation;  $\mu, \mu'$  are the cosines of the polar angles measured

\* This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

\*\* Also at Department of Meteorology, University of California at Los Angeles.

† Senior Postdoctoral Resident Research Associate of NRC-NASA. On leave of absence from Osmania University, Hyderabad, India.