

*Nonlinear Perturbations of Linear Elliptic Boundary Value Problems at Resonance**

E. M. LANDESMAN & A. C. LAZER

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1. Introduction and preliminaries. Let D be a bounded domain of R^n . Let

$$L = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} a^{ij} \frac{\partial}{\partial x_j}$$

be a second order, self adjoint, uniformly elliptic operator on D , i.e. $a^{ij} = a^{ji}$ is assumed to be a real bounded and measurable function defined on D for $i, j = 1, \dots, n$. Furthermore, there exists a number $c > 0$ such that for all $x \in D$ and arbitrary real numbers ξ_1, \dots, ξ_n

$$(1.1) \quad \sum_{i,j=1}^n a^{ij} \xi_i \xi_j \geq c \sum_{i=1}^n \xi_i^2.$$

In this paper we consider the existence problem for generalized solutions of the boundary value problem

$$Lu + \alpha u + g(u) = h(x), \quad u(x) = 0 \quad \text{on} \quad \partial D.$$

Here h is a real function in $L^2(D)$, α is a positive constant, and g is a real valued function which is bounded and continuous on the real line and satisfies an additional restriction given below. As the title of this paper indicates we assume the existence of generalized solutions for the linear homogeneous problem

$$(1.2) \quad Lu + \alpha u = 0, \quad u(x) = 0 \quad \text{on} \quad \partial D,$$

and do not necessarily assume that h is orthogonal to the solutions of (1.2) with respect to the usual $L^2(D)$ inner product.

Before stating our result we recall briefly some fundamental results and definitions from the Hilbert space theory of linear elliptic boundary value problems.

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