

On a Certain Type of Commutator

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Introduction. Let H be a separable infinite dimensional Hilbert Space. The class of commutators $AB - BA$ where A is a (bounded linear) Hermitian operator and B is an arbitrary (bounded linear) operator on H will be denoted by X_H . The subclass of commutators of X_H for which B is of the form iD where D is Hermitian will be denoted by Y_H . One can show easily that Y_H is identical with the class of Hermitian operators in X_H . The following proposition was proved by H. Radjavi in [5]:

(*) *The following three conditions are necessary and sufficient for a Hermitian operator C to belong to Y_H .*

a) *There exists an infinite orthonormal sequence $\{e_i\}$ in H such that $|\sum_{i=1}^n (Ce_i, e_i)|$ is bounded.*

b) *C is not of the form $C_1 \oplus C_2$ where C_1 has finite dimensional domain and C_2 satisfies the condition*

$$\inf_{\|x\|=1} |(C_2x, x)| > 0.$$

c) *0 is in the convex hull of the set of limit points of $\text{sp } C$.*

We recall that a number z is a limit point of the spectrum of a normal operator C if either z is a point of accumulation of the spectrum in the topological sense or z is an eigenvalue of C with infinite multiplicity.

A slight change in the proof of the necessity of condition a) in [5] shows that this condition is satisfied by all operators in X_H .

It will be shown in Section 1 of this paper that b) is also necessary for an operator to be in X_H and that c) is necessary for a normal operator to belong to X_H .

In Sections 2, 3 and 4 sufficient conditions for operators to belong to $X_{H \oplus H \oplus H \oplus \dots}$, $X_{H \oplus H}$ and X_H will be given. The main result of these sections is

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