

On the Initial Value Problem for an Incompressible Fluid with Nonlinear Viscosity

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1. Introduction. The problem of setting equations of motion for a fluid is old and dates back to Bernoulli and Euler. Cauchy [3] introduced the notion of contact forces. By imposing natural axioms that define the term "fluid" he obtained the following equations of motion:

$$(1.1) \quad \rho \frac{du}{dt} = \operatorname{div} S(u) + f,$$

where $u(x, t)$ is a vector representing the fluid's velocity, S is the symmetric stress tensor and $f(x, t)$ is the applied (external) force. ρ is the density and d/dt is the total derivative. (1.1) is supplemented by the equation of continuity

$$(1.2) \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho u) = 0.$$

In order to be able to pose a meaningful problem, additional hypotheses are necessary. Stokes [4] related the stress tensor S to the deformation tensor D .

$$(Du)_{i,i} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_i}{\partial x_i} \right),$$

where $u_i(x, t)$ are the components of $u(x, t)$.

He required that

$$(1.3) \quad S = F(D) - p(x)I,$$

where F is a continuous function and the relation (1.3) is tensorial, *i.e.* F is invariant under orthogonal transformations. He assumed also that if $Du = 0$ then $S = -p(x)I$. The scalar $p(x)$, called "the pressure," is not related to D ;

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