

The Prevalence of Uniquely Ergodic Systems

ROBERT I. JEWETT*

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1. Introduction. In the theory of dynamical systems, the relationship between topological and measure-theoretic properties is of some interest. Also of interest would be the lack of any relationship. The following theorem asserts that, for a dynamical system, the lone fact of being uniquely ergodic has almost no measure-theoretic consequences.

Theorem. *Let there be given a weakly mixing measure-preserving invertible transformation of a Lebesgue space. Then this ergodic system is equivalent to a dynamical system consisting of a homeomorphism of the Cantor Set onto itself, where the homeomorphism leaves invariant precisely one probability Borel measure.*

It seems likely that if the condition of being weakly mixing were replaced by that of being ergodic, the theorem would still be valid.

Several definitions and references will be given here. The remainder of the paper is devoted to a proof of the theorem.

A *uniquely ergodic* dynamical system is a pair (X, T) where X is a compact Hausdorff space and T is a homeomorphism of X onto itself which leaves invariant precisely one regular probability Borel measure. In this case, T is necessarily ergodic with respect to the measure.

A *Lebesgue space* is a measure space which is isomorphic to $(I, \mathfrak{B}, \lambda)$ where I is the closed unit interval, \mathfrak{B} the Borel sets, and λ Lebesgue measure. Most familiar continuous probability measure spaces are, in fact, Lebesgue spaces.

A measure-preserving transformation T of a measure space (X, \mathfrak{A}, μ) is *weakly mixing* if the only complex-valued measurable functions f on X satisfying an equation of the form $fT = \alpha f$, for α complex, are the essentially constant functions.

Two measurable systems $(X, \mathfrak{A}, \mu, T)$ and $(Y, \mathfrak{B}, \nu, U)$ are said to be *equivalent* if there exist sets A in \mathfrak{A} , B in \mathfrak{B} , and an invertible one-to-one measure-preserving mapping $S : A \rightarrow B$ such that $\mu(A) = 1$, $\nu(B) = 1$, $T(A) = A$, $U(B) = B$, and $U = STS^{-1}$.

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