

Cohomology Theory for Banach Manifolds*

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The object of this paper is to introduce a cohomology theory for the category of certain smooth Banach manifolds and their proper, smooth Fredholm maps. This theory seems to be an appropriate generalization of Čech cohomology with compact supports. In particular there are versions of Poincaré and Alexander duality valid for this theory and hence one obtains separation theorems *à la* Jordan–Brouwer. We also introduce a generalization of the finite dimensional Brouwer degree which includes as a special case the Leray–Schauder degree of compact perturbations of the identity. We also mention a few applications including a Lefschetz coincidence theorem.

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1. Preliminary results. All manifolds will be paracompact, Hausdorff and C^∞ -smooth; the model being a Banach space admitting a norm which is C^∞ except at the origin. $GC(E)$ will be the *Fredholm group* of E [6] *i.e.* the subgroup of $GL(E)$ consisting of elements of the form (Identity + compact operator). If E is real, $GC(E)$ has two components distinguished by the Leray–Schauder index; the identity component will be denoted $GC^+(E)$, the other component will be denoted $GC^-(E)$. If E is complex $GC(E)$ is connected.

Let M be a C^∞ -manifold modeled on E . A *Fredholm structure* on E is a maximal atlas $\{(U_\alpha, \varphi_\alpha)\}_{\alpha \in A}$ of M such that for any $x \in U_i \cap U_j$, $D(\varphi_i \circ \varphi_j^{-1})(\varphi_j(x)) : E \rightarrow E$ is an element of $GC(E)$. A *Fredholm manifold* is a manifold with a Fredholm structure. Thus the principal bundle $\pi_M : PM \rightarrow M$ of a Fredholm manifold is a principal $GC(E)$ -bundle. If π_M admits a reduction to $GC^+(E)$ we say that M has an *orientable structure*, a choice of such a reduction is called an *orientation*.

Now reductions of π_M to $GC^+(E)$ correspond in (1 – 1)-fashion with sections of:

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