

*The Mathematical Problem of Hydrodynamic Stability**

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1. Introduction. The Navier-Stokes equations governing viscous incompressible flow are

$$(1.1) \quad \begin{aligned} \partial_i u_i + u_i \partial_i u_i &= -\partial_i p + \frac{1}{R} \Delta u_i, \\ \partial_i u_i &= 0, \end{aligned}$$

where $\partial_i = \partial/\partial x_i$, $\partial_t = \partial/\partial t$, and repeated indices denote summation. The functions u_i , $i = 1, 2, 3$, are the components of the velocity field in rectilinear coordinates, and p denotes the pressure. The quantities in (1.1) are non-dimensional. The parameter R is called the Reynolds number for the flow and is a dimensionless quantity.

Let D be a bounded region of \mathbf{R}^3 with boundary ∂D . Due to the viscosity, the fluid must adhere to the walls of D . If the boundaries are moving at a constant, tangential rate (such as a sphere or cylinder rotating about its axis of symmetry) then the fluid must move with the walls at the boundary. Let \tilde{u} be a stationary (time independent) solution of (1.1) satisfying non-homogeneous boundary conditions. An interesting question, and one which has occupied an important position in the research of experimentalists and applied mathematicians, is that of the *stability* of the flow \tilde{u} . That is, if the flow is disturbed slightly, do the perturbations grow or decay? If the solution of (1.1) is written in the form $u = \tilde{u} + v$, where v denotes the perturbed flow, then we derive for v the system of equations

$$(1.2) \quad \begin{aligned} \partial_i v_i + v_i \partial_i v_i + \tilde{u}_i \partial_i v_i + v_i \partial_i \tilde{u}_i &= -\partial_i p + \frac{1}{R} \Delta v_i, \\ \partial_i v_i &= 0, \quad v_i = 0 \quad \text{on} \quad \partial D. \end{aligned}$$

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