

*Existence and Uniqueness of Solutions of a Class of Nonlinear Elliptic Boundary Value Problems**

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§1. Introduction. This paper was principally motivated by a number of recent studies [1–5] of a nonlinear eigenvalue problem arising in nuclear reactor theory. One is interested in positive solutions of

$$(1.1) \quad \begin{aligned} \Delta \psi + B^2 \psi &= -S(x) + c\psi^2, & x \in D, \\ \psi &= 0, & x \in \partial D, \end{aligned}$$

where B^2 and c are positive physical constants and $S \geq 0$. The set D is a suitably restricted domain in m dimensional space and $x = (x_1, x_2, \dots, x_m)$; $m \leq 3$.

It is helpful to try to obtain some physical insight into the problem. Suppose both S and c are zero. Then (1.1) has a positive solution if and only if the reactor system is just critical. If a source S is added in this situation the system becomes supercritical and a physically meaningful (*i.e.* positive) ψ is not possible. However, when the nonlinear term is *also* introduced it acts as a sink. If, physically, the flux ψ builds up, the effects of the sink exceed those of the source and it is reasonable to suspect that the system will “run.”

Indeed, even if B is such that the source- and sink-free system is supercritical it seems quite likely that the presence of the non-linear sink term will compensate for both the source and the size of B and that a positive solution ψ will exist.

Now suppose the homogeneous system is subcritical. The physical system will not operate, and surely the addition of a sink (without a source) cannot result in its running. On the other hand, the addition of a source and no sink will produce a physically meaningful ψ . Finally, if both a source and sink are added to the subcritical system it is reasonable to expect the system to operate, since the sink effect remains small provided the flux ψ does not get too large.

The existence of positive solutions of (1.1) in the case $S > 0$, $c > 0$, has been shown by Cohen [3]. He has also demonstrated uniqueness in the subcritical

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