

Roots of Ergodic Transformations

M. A. AKCOGLU & J. R. BAXTER

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Let (X, \mathfrak{F}, μ) be a finite Lebesgue space, *i.e.* a measure space isomorphic to a finite interval of the real line. The purpose of this paper is to prove the following theorem:

(1) Theorem. *Let Q be any set of positive primes. Then there exists an automorphism τ of (X, \mathfrak{F}, μ) such that:*

(i) τ is weakly mixing,

(ii) for any positive integer n , there exists an automorphism σ of (X, \mathfrak{F}, μ) such that $\sigma^n = \tau$, if and only if no positive prime in Q^c divides n .

This result answers a question raised by Friedman [6], and generalizes a result of Chacon [3], [4], showing the existence of a weakly mixing automorphism without roots of any order. A similar theorem, in the discrete spectrum case, is due to Blum and Friedman [2].

The paper consists of three sections. In the first section some preliminaries are given and the construction of τ is described. In the second section it is shown that if p is any positive prime in Q^c then τ does not have a p th root. The methods used in this section are somewhat modified versions of those used by Chacon in [3], [4]. Finally in the third section it is proved that if n is a product of powers of primes in Q then τ has an n th root.

1. Definitions and preliminaries. Let τ be an automorphism of X ; *i.e.* let $\tau: X \rightarrow X$ be an invertible transformation, such that τ and τ^{-1} are measurable and measure preserving. As usual, no distinction will be made between transformations that agree almost everywhere with respect to μ . τ defines a linear operator U_τ on measurable functions by the equation $U_\tau f(\tau x) = f(x)$, for every $x \in X$.

The automorphism τ to be constructed will have the following property: if σ is an automorphism such that $\sigma\tau = \tau\sigma$, then a sequence $n(k)$ of positive integers can be found, such that $\lim_{k \rightarrow \infty} U_\tau^{n(k)} f = U_\sigma f$ for every $f \in \mathcal{L}_p(d\mu)$, $1 \leq p < \infty$. For brevity, this property will be referred to as property *A*. The fact that the