

# *The Class $\mathfrak{W}$ of Operator-Valued Weight Functions*

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*Communicated by M. ROSENBLATT*

**1. Introduction.** In this paper we study the class  $\mathfrak{W}$  of weight functions introduced by Helson and Sarason [5] in the scalar case and by Rabindranathan [8] in the operator-valued case. Helson and Sarason gave necessary and sufficient conditions for a weight function to be in  $\mathfrak{W}$  when the related shift is of multiplicity one. Rabindranathan's conditions are valid even when the coefficient space is infinite dimensional, but Rabindranathan restricts himself to the consideration of weight functions that are bounded above and bounded away from zero.

In Section 2 below we indicate that Rabindranathan's Theorem is a corollary to a generalized Nehari Theorem for vectorial Hankel operators ([7], Theorem 5). In Section 3, we apply a recent theorem of Sz.-Nagy and Foias concerning dilations of commuting operators to remove Rabindranathan's restrictions.

**2. Preliminaries.** Let  $C$  denote a separable, complex Hilbert space, and  $m$  normalized Lebesgue measure on  $[-\pi, \pi]$ . Then  $L_C^2$  is the Hilbert space of all weakly measurable functions from the unit circle to  $C$  having square-integrable norm, and the inner product in  $L_C^2$  is

$$\langle f, g \rangle = \int_{-\pi}^{\pi} \langle f(e^{it}), g(e^{it}) \rangle dm(t).$$

The Hardy subspace of  $L_C^2$  is  $H_C^2$ . We let  $\chi$  denote the identity function on the unit circle so that the unilateral shift  $S$  on  $H_C^2$  is given by  $S: f \rightarrow \chi f$ . The above is discussed in detail by Helson [4].

The algebra of bounded operators on  $C$  is denoted by  $B(C)$ , and the algebra of weakly measurable, essentially bounded functions from the unit circle to  $B(C)$  is denoted by  $L^\infty[B(C)]$ . With the sup norm  $L^\infty[B(C)]$  is a Banach algebra. A weakly measurable function with values in  $B(C)$  will be called *integrable* if its pointwise norm, which is necessarily a measurable function since  $C$  is separable, is integrable. Such a function  $B$  is said to be *analytic* if