

# *Wiener Integral Representations for Certain Semigroups which have Infinitesimal Generators with Matrix Coefficients*

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**1. Introduction.** Let  $B_j$  be a bounded continuous  $N \times N$  complex *matrix-valued* function on  $\mathbf{R}^n$  for  $j = 1, 2, \dots, n$  and let  $V$  be a bounded Borel measurable  $N \times N$  complex *matrix-valued* functions on  $\mathbf{R}^n$ . Finally let  $\partial_i = \partial/\partial x^i$ ,  $B \cdot \nabla = \sum_{i=1}^n B_i \partial_i$ , and  $\Delta = \sum_{i=1}^n (\partial_i)^2$ . It is the purpose of this paper to prove that

$$(1.1) \quad \exp(t[\frac{1}{2}\Delta + B \cdot \nabla + V])\varphi(x) = \mathbf{M}_x(\alpha(t)\varphi(X_t)),$$

where  $\{X_t\}$  is the Wiener process in  $\mathbf{R}^n$ ,  $\mathbf{M}_x$  is the Wiener integral for paths starting at  $x \in \mathbf{R}^n$ ,  $\varphi \in L_2(\mathbf{R}^n; \mathbf{C}^N)$ , and  $\alpha(\cdot)$  is the solution of the stochastic integral equation

$$(1.2) \quad \alpha(t) = I + \int_0^t \alpha(\tau)B(X_\tau) dX_\tau + \int_0^t \alpha(\tau)V(X_\tau) d\tau.$$

(See Remark 2.3 where the precise meaning of the left hand side of (1.1) is given.)

Results similar to those in this paper have been obtained by Daleckiĭ [6] although the method of proof is completely different. For the scalar case ( $N = 1$ ) the solution (1.2) can be given explicitly as follows:

$$(1.3) \quad \alpha(t) = \exp \left[ \int_0^t B(X_\tau) dX_\tau + \int_0^t (V(X_\tau) - \frac{1}{2}B^2(X_\tau)) d\tau \right].$$

(See Theorem 7.2, p. 222, in [8]. Also see [5] and references given there to earlier papers of Cameron and Martin.)

Using (1.3) in the right hand side of (1.1) we obtain the well-known Wiener integral representation for solutions of diffusion equations. See, for example,

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