

*Nonlinear Sturm-Liouville Eigenvalue Problems and Topological Degree**

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§1. In this paper, we study boundary value problems of the form

$$(1.1) \quad \begin{cases} Lu = \lambda(a(x) - f(x, u, u'))u, & 0 < x < 1, \\ \alpha_0 u(0) + \beta_0 u'(0) = 0, & \alpha_1 u(1) + \beta_1 u'(1) = 0, \end{cases}$$

where L is the linear Sturm-Liouville operator

$$(1.2) \quad Lu = -(pu')' + qu.$$

We are interested in the question of for what values of $\lambda > 0$ there exist non-trivial solutions of (1.1).

Assuming

$$(f_1) \quad f \text{ is continuous on } [0, 1] \times \mathbf{R}^2 \text{ and } f(x, 0, 0) = 0 \text{ for } 0 \leq x \leq 1,$$

and linearizing (1.1) about $u = 0$ (with λ held fixed) yields the linear Sturm-Liouville problem

$$(1.3) \quad \begin{cases} Lv = \lambda av, \\ v \in B.C., \end{cases}$$

where $B.C.$ denotes the set of once continuously differentiable functions which satisfy the boundary conditions of (1.1). Assuming (1.3) is a regular Sturm-Liouville problem, there is a sequence $\lambda_1 < \lambda_2 < \lambda_3 < \dots$ of simple eigenvalues and a corresponding sequence v_1, v_2, v_3, \dots of (normalized) eigenfunctions such that the pairs (λ, v) satisfying (1.3) are of the form $(\lambda_k, \alpha v_k)$. The eigenfunction v_k is characterized by the fact that it has exactly $k - 1$ simple zeros (nodes) on $(0, 1)$ (see *e.g.* [1]).

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