

# *On Nonlinear Singular Perturbation Problems with Interior Nonuniformities*

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**1. Problem Statement.** Let us consider certain two-point boundary value problems of the form

$$(1.1) \quad \begin{cases} \epsilon y'' = f(x, y, y', \epsilon), & 0 \leq x \leq 1 \\ y(0) = \alpha(\epsilon), \quad y(1) = \beta(\epsilon), \end{cases}$$

where the "reduced problem"

$$(1.2) \quad \begin{cases} f(x, z(x), z'(x), 0) = 0, & 0 \leq x \leq 1 \\ z(0) = \alpha(0), \quad z(1) = \beta(0), \end{cases}$$

has an "angular solution"

$$(1.3) \quad z(x) = \begin{cases} g_0(x), & 0 \leq x \leq x_0 \\ h_0(x), & x_0 \leq x \leq 1, \end{cases}$$

where

$$g_0(x_0) = h_0(x_0) \equiv y_0,$$

and

$$\mu_1 \equiv g'_0(x_0) \neq h'_0(x_0) \equiv \mu_2.$$

We shall assume that  $f$  is infinitely differentiable in all its arguments, that  $\epsilon$  is a small positive parameter, and that  $\alpha(\epsilon)$  and  $\beta(\epsilon)$  have prescribed asymptotic expansions, say

$$(1.4) \quad \alpha(\epsilon) \sim \sum_{i \geq 0} \alpha_i \epsilon^i \quad \text{and} \quad \beta(\epsilon) \sim \sum_{i \geq 0} \beta_i \epsilon^i$$

as  $\epsilon \rightarrow 0$ . Without loss of generality, we shall take  $\mu_1 < \mu_2$ .

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