

Idempotent Probability Measures on Locally Compact Abelian Semigroups

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1. Introduction. The study of idempotent measures on topological groups and topological semigroups has been an interesting subject in the past years. In 1954 Wendel [8] started the problem by showing that the positive idempotent measures on a compact group were precisely the normed Haar measures of compact subgroups. In 1959 Rudin [5] showed that the same result held for locally compact abelian groups. In the same year Glicksberg [1] extended the result to semigroups. He showed that a positive idempotent measure on a compact abelian semigroup was the normed Haar measure on a compact subgroup. Later Pym [4] (1962) and Heble and Rosenblatt [3] (1963) showed independently that a positive idempotent measure on a compact semigroup was a product measure on a completely simple subsemigroup. In [4] Pym also extended Wendel's result to locally compact groups. In this paper, we shall generalize Glicksberg's result to locally compact abelian semigroups. Whether Pym–Heble–Rosenblatt's result can be generalized to locally compact semigroups is still unknown [6].

Besides, our indebtedness to Heble and Rosenblatt's paper will be self-evident.

2. Preliminaries. Let S denote a locally compact (Hausdorff) semigroup. Let μ and ν be two bounded regular (Borel) measures on S . The convolution $\mu * \nu$ of μ and ν is defined as the regular measure generated by the linear functional

$$L(f) = \int_S \left[\int_S f(xy) \mu(dx) \right] \nu(dy) = \int_S \left[\int_S f(xy) \nu(dy) \right] \mu(dx)$$

for $f \in C_c(S)$, the space of all continuous functions on S with compact support. The fact that the two iterated integrals are equal was shown in [2, p. 207].

Lemma 1. *Let $\mathfrak{B}(S)$ be the class of all Borel subsets of S . Suppose μ and ν are two bounded regular measures on S . Then:*

- (i) *For each $s \in S$, the measure $\mu(As^{-1})$, $A \in \mathfrak{B}(S)$, is regular on S .*
- (ii) *For each $A \in \mathfrak{B}(S)$, $\mu(As^{-1})$ is a Borel measurable function of s . In particular, if V is open, then $\mu(Vs^{-1})$ is a lower semicontinuous function of s .*