

*The Lichtenstein-Gersgorin Integral Equation in Conformal Mapping of Variable Domains**

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Introduction. The Lichtenstein-Gersgorin integral equation [9, 10], which has been used for the effective determination of conformal maps [1, 9, 4, 5], can also be employed with great advantage for the study of the variation of the mapping function under deformations of the domain. In a recent paper [10] the writer studied such a variation of the mapping function of a Jordan domain onto a disk by means of a perturbation of this integral equation and derived several applications. This perturbation method is particularly adapted to "infinitesimal" variations of the domain, as the applications to the n -th order extensions of the Hadamard and Julia formulae given in that paper indicate.

In the present paper we use this integral equation to obtain estimates for the difference of the Riemann mapping functions of two "neighboring" domains onto the unit disk and particularly for that of their derivatives of any order, which hold uniformly in the closure of (the appropriate component of) the intersection of the two domains. This yields also estimates for the difference of the inverse functions and their derivatives which are valid in the closed unit disk. (In [10] we mention on p. 67 the possibility of obtaining estimates of this type by use of the perturbation theorem; this detour is however more complicated and seems to lead to less precise results.)

Let $C_0 : \{z \mid z = \zeta_0(t), t \in [0, 1]\}$ denote a closed Jordan curve where $\zeta_0(t) \in \text{Lip}(\alpha)$, $0 < \alpha \leq 1$, in $[0, 1]$. (It is understood that $\zeta_0(t)$ and $\zeta(t)$ below are periodic with period 1.) Suppose $w = f_0(z)$ maps the Int $(C_0) = \Omega_0$ conformally onto the disk $\{w \mid |w| < 1\}$ with $f_0(z_0) = 0$, $z_0 \in \Omega_0$; $f_0(\zeta_0(t))$ is defined and continuous and $\theta_0(t) = \arg f_0(\zeta_0(t))$ satisfies the Lichtenstein-Gersgorin integral equation

$$(1) \quad \theta_0(s) = \int_0^1 K_0(s, t) \theta_0(t) dt + \beta_0(s),$$

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