

Solutions of the n -Dimensional Wave Equation in the Exterior of the Characteristic Cones (I)*

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Summary. The notion of the hyperboloidal means (HM) is introduced such that it is invariant with respect to the Lorentz and similarity transformations and satisfies the wave equation. In the exterior of the characteristic cones C_0 , the even and odd solutions which carry smooth initial data can be expressed in terms of HM of the matching boundary data on C_0 . This is proved by establishing two identities. They are put subsequently in a "weak" form applicable to L_2 -data. By approximation, the result is extended to L_2 -solutions: Using HM of the boundary values on C_0 one has a simple representation of L_2 -solutions as distributions in the exterior of C_0 ; the uniqueness of the solutions and their domains of dependence are obtained. The space dimension of the wave equation is assumed to be odd.

Introduction. The paper extends the result of a previous work [1] on the wave equation of three space dimensions to that of higher odd dimensional space. We consider smooth solutions and also L_2 -solutions $w(X, t)$ of

$$\square^2 w \equiv \sum_{i=1}^n w_{x_i x_i} - w_{t t} = 0,$$

where (X, t) is a point in the space-time $R_{X,t}^{n+1}$ and lies in the exterior of the double cone C_0 given by $t^2 - r^2 = 0$ with $r = |X|$, $X = (x_1, x_2, \dots, x_n) \in R_X^n$, n odd. Representations of $w(X, t)$ are obtained in terms of the data on C_0 and the domain of the dependence will consist of the intersection $C_0 \cap C(X, t)$ where $C(X, t)$ is the characteristic double cone with the vertex at (X, t) .

Let C_0^+ and C_0^- be respectively the upper ($t \geq 0$) and the lower ($t \leq 0$) part of C_0 . We shall consider them *separately* by projecting each cone onto the n -dimensional space R_Y^n ($Y = (y_1, y_2, \dots, y_n)$, $|Y| = \rho$) by the maps

$$(0.1) \quad P_{\pm} : (X_0, t_0) \in C_0^{\pm} \rightarrow Y = X_0 \in R_Y^n.$$

Data on C_0^+ (and C_0^-) may then be considered as being prescribed on R_Y^n . In

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