

Cauchy Problem in S'

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1. Introduction. The *Cauchy problem* is to “solve” a given system of partial differential equations,

$$(1.1) \quad \frac{\partial u_j(x, t)}{\partial t} = \sum_{k=1}^N P_{jk}(D)u_k(x, t), \quad j = 1, 2, \dots, N, \quad t > 0,$$

subject to the initial conditions,

$$(1.2) \quad \lim_{t \downarrow 0} u_j(x, t) = u_j^0(x), \quad j = 1, 2, \dots, N.$$

t is a real variable, $u_j^0(x)$ are given functions of x , and $p_{jk}(D)$ ($j, k = 1, \dots, N$) is a linear partial differential operator of order $\leq p$ with constant coefficients, *i.e.*,

$$p_{jk}(D) = \sum_{|\alpha| \leq p} p_{\alpha_1, \dots, \alpha_n}^{jk} D_{x_1}^{\alpha_1} \cdots D_{x_n}^{\alpha_n}, \quad |\alpha| = \alpha_1 + \cdots + \alpha_n, \quad D_{x_i} \equiv -i \frac{\partial}{\partial x_i}.$$

The $u_j(x, t)$ are functions of (x, t) , $x = (x_1, x_2, \dots, x_n) \in X = R_s \times T^{n-s}$, the Cartesian product of s -dimensional Euclidean space and the $n - s$ torus $T^{n-s} \equiv R_{n-s}/Z^{n-s}$, ($Z \equiv$ additive group of integers).

The system (1.1), (1.2) may be rewritten in vector form,

$$(1.3) \quad \frac{\partial u(x, t)}{\partial t} = P(D)u(x, t), \quad t > 0,$$

$$(1.4) \quad \lim_{t \downarrow 0} u(x, t) = u_0(x).$$

Despite the recent trend, *cf.* [11], of treating space and time coordinates equally, the time coordinate will be given a preferred role. The equations are such that if a “state” of a given system is specified at some initial time $t = 0$ then a “solution” exists for $t > 0$ and is uniquely determined by the equations together with side conditions, which usually take the form either of boundary conditions or of growth conditions at infinity. The state of the system at any time t is given by a finite set of functions, denoted by $u(t)$, which depend *only* on the space coordinates. As time progresses, the state of the system changes

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