

Collapse of Thick Spherical Shells by External Pressure

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1. Introduction. Many experiments were made by P. W. Bridgman [1] with respect to the plastic flows and fractures under conditions of high stress. Especially he found that the relations between the external collapsing pressure and the geometrical configuration of a deformed thick hollow cylinder. T. Y. Thomas [2] attacked this problem theoretically and obtained the remarkable results which show the identical relations observed by Bridgman.

In this paper, by the similar method of Thomas [2], the plastic flow deformations of the spherical shells by the external pressure are analysed and the stress field and the collapsing pressure are determined. The following result is found: The logarithm of the ratio of outer and inner radii just prior to collapse is a linear function of the collapsing pressure. By the slope of the graphical line and its intercept with the ordinate axis in the above linear relation, two parameters representing the magnitude of work-hardening effect are calculated.

2. Basic relations. If the external pressure is not so high, the entire region of a spherical shell remains on elastically deformed state. Denoting by x^1, x^2, x^3 spherical coordinates r, θ, ϕ , the stress components are given by

$$(2.1) \quad \begin{cases} \sigma_{(1)}^{(1)} = -\frac{p}{1 - a^3/b^3} \left(1 - \frac{a^3}{r^3}\right), \\ \sigma_{(2)}^{(2)} = \sigma_{(3)}^{(3)} = -\frac{p}{1 - a^3/b^3} \left(1 + \frac{a^3}{2r^3}\right), \\ \sigma_{ij} = 0 \quad (i \neq j), \end{cases}$$

where p is the pressure, a and b are inner and outer radii of the spherical shell, respectively, and $\sigma_{(i)}^{(i)} = \sigma_i^i (g_{ii}/g_{ii})^{1/2}$ (not summed on i and j) are the physical components of the stress state and we have

$$(2.2) \quad \sigma_{(1)}^{(1)} = \sigma_{11}, \quad \sigma_{(2)}^{(2)} = \frac{\sigma_{22}}{r^2}, \quad \sigma_{(3)}^{(3)} = \frac{\sigma_{33}}{r^2 \sin^2 \theta}.$$