

# *Factorization of Operator Valued Entire Functions*

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**1. Introduction.** Let  $w$  be a complex valued entire function of exponential type  $\tau$  that is not identically zero. Exponential type is defined as in Boas [2]. If  $w$  has non-negative values on the real axis and

$$(1) \quad J(w) = \int_{-\infty}^{\infty} (1 + x^2)^{-1} \log^+ w(x) dx < \infty,$$

then ([2, p. 125], [3, p. 34], [6, p. 437])

$$w = f^*f$$

where  $f$  is an entire function of exponential type  $\frac{1}{2}\tau$  that has no zeros in the upper half-plane  $y > 0$ . For any entire function  $f$ ,  $f^*$  is defined by  $f^*(z) = \bar{f}(\bar{z})$  for all complex  $z$ . In this paper we prove an analogous theorem for operator valued entire functions. We deduce as a corollary a factorization theorem for polynomials with operator coefficients which have non-negative values on the real axis.

A novel aspect of our work is the use of abstract shift and Toeplitz operator theory to prove an abstract Hilbert space theorem (Theorem 3) that is the key to the factorization theory. This is done in Section 3, and the main theorem is there proved under the assumption that the given function is bounded on the real axis. Section 2 is devoted to statements of our main results. In Section 4 we review the function theory that is needed elsewhere in the paper. In Section 5 we prove the main theorems. A strengthened form of the factorization theorem is derived in Section 6.

Our results were announced in [10]. For clarity of exposition we have chosen to limit our main discussion (Sections 2-5) to an essential case of the announced factorization theorem, leaving the general case to Section 6. Our treatment does not follow a direct path to the factorization theorem. Rather, we have intended to emphasize the algebraic structure underlying the proof.

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